Computing an Aggregator's Long Term Profit under Uncertain Behavior of the Agents

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Abstract

In the retail electricity market, consumers can subscribe a contract with a conventional retailer or cooperate through an aggregator who takes forward positions in the wholesale electricity market, modeled as a two-tiered system. We characterize analytically the core of the game and give conditions for its non emptiness. Then we propose a Machine Learning algorithm to forecast the consumers' demand and use these forecasts as inputs to optimize the aggregator's pricing strategy. The viability of the aggregator's pricing strategy is finally evaluated on a case study containing the power consumptions of 370 Portuguese consumers over four years.

Keywords: Sequential Games; Core; Hierarchical Learning

1 Introduction

Up to now, conventional consumers have a contractual relation with a retail electricity provider that supplies them electricity at a price defined through contract mechanisms (such as flat rate, HP/HC, Blue/White/Red, Time of Use, etc.). The arrival of aggregators could encourage consumers to become smarter. In France, various business models for the aggregator coexist: for example, the electricity provision from Enercoop relies exclusively on a cooperative network of local renewable producers; Grid Pocket uses behavioral economics and the analysis of the consumers' data patterns to optimize his clients' daily power consumption so as to reduce their energy bill; Energy Pool also aims at reducing his clients' energy bill but through the intelligent scheduling of the daily loads.

In this paper, smart consumers are members of a smart load balancing group that is managed by an aggregator. The group size might evolve dynamically and will be denoted $\mathcal{G}(t)$ at period t. The aggregator buys electricity in the day-ahead (wholesale) electricity market. From the market operator point of view, the aggregator is a single buyer of electricity. In our model, the aggregator takes forward positions in the day-ahead market which coincide with the aggregation

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of his clients' forward positions. If real-time consumption differs from day-ahead estimation, the aggregator, and through him the smart coalition of consumers, is charged for this difference at a higher unit price. However, the aggregator might compensate overestimation errors of his clients by underestimation and vice versa. He might make profit through two different ways: firstly, by his classical activity of selling energy which gives rise to an activity profit. Secondly, by following the covering mechanism described above, leading to a covering profit. The sum of both gives the aggregator's total profit.

We define a *coalition* of consumers as a set of end users who agree on a joint demand profile to be contracted in the wholesale electricity market with the mediation of an aggregator. We let $\mathcal{N}$ be the set containing all the consumers (both the aggregator's and the conventional retailer's clients), of cardinality $N \in \mathbb{N}^*$.

Over each day, each consumer has exactly one base load, that cannot be shifted, and several shiftable loads. The scheduling of the shiftable loads, which is out of the scope of this paper, might be based on decentralized control [3], priority mechanisms or economic incentives such as rebates. Under this latter mechanism, consumers can be refunded provided they agree to defer their loads in case of peak demands to fill valleys where power consumption is lower.

In this paper, we assume that only the base load may differ from the estimated one since all the shiftable loads are assumed to be scheduled automatically [5] and focus exclusively on the base load management over a long term period such as months.

We start by describing the interactions between the aggregator and the electricity market, utility functions and judgment accuracy issues in Section 2. In Section 3, we give our main analytical results. Finally, in Section 4, the viability of the aggregator's pricing strategy is evaluated in a case study.

2 Utility functions and uncertainty representation

We let $p^f(t)$ be the day-ahead wholesale market price and $p^0(t)$ be the real-time wholesale market price at period t. In the balancing, we introduce $p^+(t)$, the unit penalty when excess power must be sold by the aggregator and $p^-(t)$, the unit penalty when more power must be purchased by the aggregator. Following the French Transmission System Operator's balancing rules[1], we have the relations: $p^0(t) < p^-(t)$ and $p^+(t) < p^0(t)$ at any period t. Smart consumer i is penalized on the basis of the difference between his estimated demand (in the day ahead) $\hat{d}_i(t)$ and the realization of his demand (in real time) $d_i(t)$ depending on whether he has chosen a long or short position. We now describe more precisely the penalty mechanism. In case of a long position, the forecast of consumer i is higher than the realization of his demand. The aggregator must sell the total excess power bought in the day-ahead market, in the balancing market, at price

[1] `http://rei.revues.org/4053` [Online September 2015]

$p^+(t)$. Consumer i is then penalized on the basis of $p^0(t) - p^+(t)$ per excess power unit leading to: $(p^0(t) - p^+(t))\Big(\hat{d}_i(t) - d_i(t)\Big)_+$[2]. In case of a short position, the forecast of consumer i is smaller than the realization of his demand. The aggregator buys the total missing power on the balancing. Consumer i is penalized on the basis of $p^-(t)$ per missing power unit leading to: $p^-(t)\Big(\hat{d}_i(t) - d_i(t)\Big)_-$. The aggregator simply transfers the cost of buying the missing power to her clients having short positions.

We assume that there is a noise deforming the welfare provided by a power unit. We let $\mu_i^*(t) \sim f(\theta_i^*)$ be the noise affecting consumer i's evaluation of the provider *'s offer where θ_i is a parameter characterizing this density function (its mean for example). We assume that at any period t, $\mu_i^{agg}(t)$ and $\mu_i^{ret}(t)$ are independent. Furthermore, noises are assumed to be independent and identically distributed (iid) between two distinct consumers and and there is no time dependence between two consecutive noises. This modeling allows us to quantify the intrinsic preferences of the agents when comparing the welfare brought by each provider.

In the following, we let: $y^{agg}(i,t) = \omega\mu_i^{agg}(t) - p^{agg}(t)$ be the surplus created by the aggregator at period t by providing any consumer i with one power unit, where $p^{agg}(t)$ is the unit price charged by the aggregator and $\omega > 0$ is the welfare created by that power unit. Symmetrically for the retailer: $y^{ret}(i,t) = \omega\mu_i^{ret}(t) - p^{ret}(t)$ denotes the surplus created by the retailer, where $p^{agg}(t)$ is the unit price charged by the aggregator. We define the utility function for consumer i depending on which provider * he subscribes, as follows: $U_i^*(t) = y^*(i,t)d(i,t)$ where $d(i,t) > 0$ is the demand of consumer i. Consumer i prefers the aggregator to the conventional retailer if, and only if $U_i^{ret}(t) < U_i^{agg}(t)$ which is equivalent to $\mu_i^{ret}(t) - \mu_i^{agg}(t) < \frac{p^{ret}(t) - p^{agg}(t)}{\omega}$. We let $\Delta\mu_i = \mu_i^{ret}(t) - \mu_i^{agg}(t)$ be the difference between the noises introduced by consumer i regarding his evaluation of the retailer and the aggregator's offers and $F_{\Delta\mu_i}$ be the associated cumulative distribution function.

3 A sequential game

$\Pi_C(t)$ is the aggregator's covering profit resulting from the covering of opposite positions among his clients and $\Pi_A(t)$ is the aggregator's activity profit: $\Pi_A(t) = \sum_{i\in\mathcal{G}(t)} d_i(t)p^{agg}(t)$.

The aggregator's objective is to maximize the expectation of his total profit over T periods: $\max \mathbb{E}\Big[\sum_{t=1}^T \Big(\Pi_A(t) + \Pi_C(t)\Big)\Big]$ such that $0 \le p^{agg}(t) \le \bar{p}, \forall t = 1, ..., T$ where $\bar{p} > 0$ is a price cap fixed on the providers' retail price.

Similarly the conventional retailer's objective is:
$\max \mathbb{E}\Big[\sum_{t=1}^T \sum_{i\in\mathcal{N}-\mathcal{G}(t)} p^{ret}(t)d_i(t)\Big]$ such that $0 \le p^{ret}(t) \le \bar{p}, \forall t = 1, ..., T$. The solving of these sequential optimization problems gives rise to sequences

[2] For any real x, we set: $x_+ \triangleq \max\{0; x\}$ and $x_- \triangleq \min\{0; x\}$

of prices $\Big\{p^*(t)\Big\}_{t=1,\dots,T}$. We now describe the game that takes place at each period:

1) Each consumer enters the decision process or remains inert according to a parameter β. When consumers are used to a certain provider for one of their needs, they will not always consider alternative options. This tendency takes into account the general preferences of economic agents to choose the status quo: putting oneself together in order to make a decision has a cost in itself.

2) The consumers who have entered the decision process, choose one provider depending on the providers' prices and of the intrinsic preferences of the consumers.

To solve the two steps game for the base loads, we will proceed backwards. One major concern regarding Step 2) is whether the optimal price fixed by the aggregator guarantees the stability of his coalition. In the following, we will characterize analytically how the aggregator should share his cost among his clients to stabilize his coalition.

Definition of the cost of the coalition: We let $\epsilon_i(t) = \hat{d}_i(t) - d_i(t)$ be the error between the forecasted demand and the real demand of consumer i. To enable analytical tractability, we assume that: $\epsilon_i(t) \sim \mathcal{N}(0;\sigma_i^2)$ is error distributed according to a Gaussian density function centered in zero and of standard deviation $\sigma_i > 0$. The $\epsilon_i(t)$ are iid, there is no time dependence between two consecutive errors and $\mu_i^*(t)$ is independent of any $\epsilon_i(t)$. In turn $\sum_{i\in\mathcal{G}(t)} \epsilon_i(t)$ is distributed according to a Gaussian density function centered in zero and of variance $\sum_{i\in\mathcal{G}(t)} \sigma_i^2 > 0$.

According to the stylized market design introduced in [2], the cost of coalition $\mathcal{G}(t)$ for the aggregator at time period t is:

$$c\Big(\mathcal{G}(t)\Big) \quad = \quad p^f(t) \sum_{i\in\mathcal{G}(t)} \hat{d}_i(t) + p^-(t)\Big(\sum_{i\in\mathcal{G}(t)} \epsilon_i(t)\Big)_- - p^+(t)\Big(\sum_{i\in\mathcal{G}(t)} \epsilon_i(t)\Big)_+ \quad (1)$$

The cooperative game that we consider is with Transferable Utility (TU) because the coalitional value/cost, $c\Big(\mathcal{G}(t)\Big)$, defined in Equation (1) can be divided amongst the aggregator's clients in any way that the aggregator's clients choose. We introduce the notion of core which characterizes the stability of the coalition made of the aggregator's clients. For TU game, the core is defined as the set of imputations $\Big\{z^{agg}(i,t)\Big\}_{i\in\mathcal{G}(t)}$ such that no consumer has an incentive to switch from the aggregator to the conventional retailer. In other words, the core is a set of joint strategies with which all consumers want to cooperate in a smart consumer group and any deviating coalition (i.e., consumers switching to the conventional retailer) cannot guarantee higher utilities to all of its members. With this respect, the core strategies are stable.

We assume that for any consumer i, the opposite of his expected utility is associated to a unique imputation: $z^*(i,t) = -\mathbb{E}\Big[U_i^*(t)\Big]$. Using $U_i^*(t)$ definition, consumer i's imputation is completely determined by the provider's price, estimated demand and game parameters: $z^*(i,t) = -\omega\theta_i^*\hat{d}_i(t) + p^{agg}(t)\hat{d}_i(t)$.

Definition of the core: The imputation $\Big\{z^{agg}(i,t)\Big\}_{i\in\mathcal{G}(t)}$ is in the core if, and only if:

$$\begin{aligned}\sum_{j\in\mathcal{G}(t)} z^{agg}(j,t) &= \mathbb{E}\Big[c\Big(\mathcal{G}(t)\Big)\Big] \quad \texttt{GR} \\ z^{agg}(i,t) &\leq z^{ret}(i,t),\ \forall i\in\mathcal{G}(t) \quad \texttt{IR}\end{aligned}$$

GR stands for Group Rationality and IR, for Individual Rationality. Our game follows a γ-model [4]: all consumers outside a coalition subscribe to the conventional retailer's offer and build single player coalitions without direct participation to the market. The game is with characteristic function $V(.,t)$ defined as follows: $V(\mathcal{G},t) \triangleq \mathbb{E}\Big[c\Big(\mathcal{G}(t)\Big)\Big]$ if $Card\Big(\mathcal{G}(t)\Big) \geq 2$ and $V(i,t) \triangleq \mathbb{E}\Big[c_{ret}(i,t)\Big], \forall i \in \mathcal{N}$ where the cost charged by the conventional retailer is $c_{ret}(i,t) = p^{ret}(t)d_i(t)$.

Non emptiness of the core: Characterizing the aggregator's coalition stability is equivalent to prove that the core of the associated TU game is non empty. To check that the core is non empty, we rely on the Bondareva-Shapley Theorem i.e., we need to prove that the TU game is balanced.

Proposition 1. *The core is non empty if, and only if:* $\min_{i\in\mathcal{N}}\Big\{\hat{d}_i(t)\Big\}$ $\geq \frac{1}{N}\frac{p^f(t)}{p^{ret}(t)}\sum_{j\in\mathcal{N}}\hat{d}_j(t) + \frac{1}{N}\frac{p^-(t)-p^+(t)}{p^{ret}(t)}\sqrt{\sum_{j\in\mathcal{N}}\sigma_j^2}$.

Proof of Proposition 1. We recall that the TU game is balanced if, and only if:

$$\sum_i \lambda_{ret}(i)\mathbb{E}[c_{ret}(i,t)] + \lambda_{agg}\mathbb{E}[c\Big(\mathcal{G}(t)\Big)] \geq \mathbb{E}[c\Big(\mathcal{G}(t)\Big)] \tag{2}$$

for all the collections of weights $\Big\{\lambda_{ret}(i)\Big\}_i, \lambda_{agg}$ such that $\lambda_{ret}(i) \in [0;1], \forall i$, $\lambda_{agg} \in [0;1]$ and $\lambda_{ret}(i) + \lambda_{agg} = 1, \forall i$. This implies that $\lambda_{agg} = 1 - \lambda_{ret}(i), \forall i$; a fortiori: $\lambda_{agg} = 1 - \frac{1}{N}\sum_i \lambda_{ret}(i)$. By substitution in Inequality (2), we infer that it is equivalent to: $\sum_i \lambda_{ret}(i)\Big\{\mathbb{E}[c_{ret}(i,t)] - \frac{1}{N}\mathbb{E}[c\Big(\mathcal{G}(t)\Big)]\Big\} \geq 0$ for all the collections of weights $\Big\{\lambda_{ret}(i)\Big\}_i, \lambda_{agg}$ defined above. According to the Bondareva-Shapley Theorem, the core of our TU game is non empty if, and only if, $\mathbb{E}[c_{ret}(i,t)] - \frac{1}{N}\mathbb{E}[c\Big(\mathcal{G}(t)\Big)] \geq 0, \forall i$. Therefore, we need to check whether this relation holds. Analytically, we have: $\mathbb{E}[c_{ret}(i,t)] - \frac{1}{N}\mathbb{E}[c\Big(\mathcal{G}(t)\Big)] = \mathbb{E}\Big[p^{ret}(t)d_i(t)\Big] - \frac{1}{N}\mathbb{E}\Big[p^f(t)\sum_j \hat{d}_j(t) + p^-(t)\Big(\sum_j(\hat{d}_j(t) - d_j(t))\Big)_- - p^+(t)\Big(\sum_j(\hat{d}_j(t) - d_j(t))\Big)_+\Big]$.

The first term of the equation can be rewritten as: $p^{ret}(t)\hat{d}_i(t)$. Furthermore we recall that any real x can be decomposed as a linear combination of its positive and negative parts: $x = x_+ - x_-$. This implies that:

$$\begin{aligned}
& \mathbb{E}[c_{ret}(i,t)] - \frac{1}{N}\mathbb{E}[c\Big(\mathcal{G}(t)\Big)] \\
= \quad & \Big(p^{ret}(t) - \frac{1}{N}p^f(t)\Big)\hat{d}_i(t) - \frac{1}{N}p^f(t)\sum_{j\neq i}\hat{d}_j(t) - \frac{1}{N}p^-(t)\,\underbrace{\mathbb{E}\Big[\sum_j(\hat{d}_j(t) - d_j(t))\Big]}_{=0} \\
+ \quad & \frac{1}{N}\Big(\underbrace{p^+(t) - p^-(t)}_{<0}\Big)\mathbb{E}\Big[\Big(\sum_j(\hat{d}_j(t) - d_j(t))\Big)_+\Big] \qquad (3)
\end{aligned}$$

Equation (3) can be rewritten as follows: $\mathbb{E}[c_{ret}(i,t)] - \frac{1}{N}\mathbb{E}[c\Big(\mathcal{G}(t)\Big)] \geq 0, \forall i \in \mathcal{N} \Leftrightarrow p^{ret}(t)\hat{d}_i(t) - \frac{1}{N}p^f(t)\sum_{j\in\mathcal{N}}\hat{d}_j(t) + \frac{1}{N}(p^+(t) - p^-(t))\sqrt{\sum_{j\in\mathcal{N}}\sigma_j^2} \geq 0, \forall i \in \mathcal{N}$. This is equivalent to:

$$\min_{i\in\mathcal{N}}\Big\{\hat{d}_i(t)\Big\} \geq \frac{1}{N}\frac{p^f(t)}{p^{ret}(t)}\sum_{j\in\mathcal{N}}\hat{d}_j(t) + \frac{1}{N}\frac{p^-(t) - p^+(t)}{p^{ret}(t)}\sqrt{\sum_{j\in\mathcal{N}}\sigma_j^2}$$

□

Fairness: The Shapley value attributes to each consumer in the coalition an imputation which is a function of its marginal contribution to the coalition. To check the axiom of Pareto optimality, the Shapley value is averaged over all the interactions that each consumer can have with other consumers in the coalition. Formally, the Shapley value associated with consumer $i \in \mathcal{G}(t)$ is: $\varphi_i(V,t) = \sum_{\mathcal{G}\subseteq\mathcal{G}(t), i\in\mathcal{G}} \frac{(Card(\mathcal{G}(t)) - Card(\mathcal{G}))!(Card(\mathcal{G})-1)!}{Card(\mathcal{G}(t))!}\Big(V(\mathcal{G}) - V(\mathcal{G} - \{i\})\Big)$. In the framework of our γ-model, it can easily be computed analytically: $\varphi_i(V,t) = \frac{1}{Card(\mathcal{G}(t))}\Big\{p^f(t)\hat{d}_i(t) + \frac{1}{\sqrt{2\pi}}\Big(p^-(t) - p^+(t)\Big)\Big(\sqrt{\sum_{j\in\mathcal{G}(t)}\sigma_j^2} - \sqrt{\sum_{j\in\mathcal{G}(t)}\sigma_j^2}\Big)\Big\}$ $+ \frac{1}{Card(\mathcal{G}(t))}p^{ret}(t)\hat{d}_i(t)$. It is straightforward to show that our TU game is not convex[3]. As such we have no guarantee that the Shapley value belongs to the core. But we now give conditions for the Shapley value to be in the core. GR being checked immediately by definition of the Shapley value, we now focus on IR i.e., $\varphi_i(V,t) \leq z^{ret}(i,t), \forall i \in \mathcal{G}(t)$ which give the following condition on the consumers' data and game parameters.

Proposition 2. *The Shapley value of the TU game belongs to the core if, and only if:*

$$\frac{\hat{d}_i(t)}{\sqrt{\sum_{j\in\mathcal{G}(t)}\sigma_j^2} - \sqrt{\sum_{j\in\mathcal{G}(t)-\{i\}}\sigma_j^2}} \leq \frac{1}{\sqrt{2\pi}}\frac{p^-(t) - p^+(t)}{p^f(t) + \Big(1 - Card(\mathcal{G}(t))\Big)p^{ret}(t)}, \forall i \in \mathcal{G}(t)$$

[3]We recall the convexity definition: let $\mathcal{G}$ and $\mathcal{G}'$ be two coalitions such that $\mathcal{G} \subset \mathcal{G}'$, $Card(\mathcal{G}) \geq 2$ and let $i \in \mathcal{N}\backslash\mathcal{G}'$, to prove that the game is convex we need to check that: $V\Big(\mathcal{G}' \cup \{i\}, t\Big) - V\Big(\mathcal{G}', t\Big) < V\Big(\mathcal{G} \cup \{i\}, t\Big) - V\Big(\mathcal{G}, t\Big)$.

Inertia and its consequences: In Step 1), inertia is captured by a parameter $\beta \in [0;1]$ which denotes, at every period, the probability for the consumer to get himself into the situation of making a decision on his energy supplier. We will assume $\beta \neq 0$ and $\beta \neq 1$, meaning we are neither in a situation of monopoly, nor in a situation of competition *à la Bertrand*. This first step will have a consequence on the aggregator's pricing policy.

The introduction of inertia changes the demand dynamics i.e., we have: $\sum_{i \in \mathcal{G}(t)} d_i(t) = (1-\beta) \sum_{i \in \mathcal{G}(t-1)} d_i(t) + \beta \sum_{i \in \mathcal{N}} d_i(t) s(i,t)$ where $s(i,t) \in [0;1]$ is the probability that at the beginning of period t consumer i enters a decision process on the choice of his energy retailer and chooses the aggregator. $s(i,t)$ can be computed analytically: $s(i,t) = \mathbb{P}\Big(z^{ret}(i,t) > z^{agg}(i,t)\Big) = F_{\Delta\mu_i}\Big(\frac{p^{ret}(t) - p^{agg}(t)}{\omega}\Big)$.

4 Simulations

Solving this sequential optimization game described in Section 3 requires to forecast the demand of each individual consumer over horizon T. To that purpose we use a hierarchical learning algorithm: At the lower level, we run in parallel three experts Support Vector Regression (SVR), Neural Network (NN) and Conditional External Regret. While SVR and NN produce point forecast, Regret produces a probabilistic forecast of the consumers' demand. We therefore sample a forecast according to the estimated density function. We observe that the NN achieves excellent performance. This can be explained by the high dependence of the data on features such as hour, day, month, season, etc. We observe that to achieve pretty good performance the Regret [1] has to be run on a larger training set than the two other algorithms. Based on the data characteristics, we implement (at the lower level) a Conditional External Regret algorithm. More precisely, we classify a priori the forecasted densities obtained through External Regret based on features such as the day, week and season. At the upper level, the aggregator runs an External Regret algorithm which associated a weight to the forecasts produced by each of the three experts. The combinations of these experts' forecasts gives the aggregator's forecast of each individual consumer demand. Note that the weights associated with each expert are updated following the rule described in [1].

We run our algorithm on a database containing the power consumption of 370 Portugese consumers that can be either residentials or industrials[4]. We take as parameters: $\omega = 1$, which enables us to normalize our surplus, and consider two scenarios $\beta = 0.3$ and $\beta = 0.9$. The power consumption of each individual is monitored every 15 minutes on a 4 year basis (to be precise, from the 1-st of January 2011 to December 31, 2014). In the data base, consumers are classified in 8 categories: retail/shopping, continuous laboring, weekly laboring, hotel businesses, catering industry, schools, logistics and others.

[4]Trindade A., artur.trindade@elergone.pt, Elergone, NORTE-07-0202-FEDER-038564, `https://archive.ics.uci.edu/ml/datasets.html` [Online September 2015]

	SVR	**NN**	**Cond. Ext. Regret**	**Agg.**
MSE	0.122	0.127	0.233	0.107

Table 1: Hierarchical Learning algorithm performance.

To test the hierarchical learning algorithm performance, we use it to forecast the individual consumption of one consumer over one month, having trained the each algorithm and the hierarchical learning algorithm over a whole year (the granularity of the measurements being of 15 minutes). We report the resulting Mean Square Errors (MSEs) in Table 1.

In Figure 1, we represent the dynamics of the providers' profit evaluated on our 24 month test set for two scenarios of inertia: $\beta = 0.3$ in (a) and $\beta = 0.9$ in (b). We observe that the aggregator's profit always remains positive for both values of inertia parameter.

5 Conclusion

We consider a two-tiered electricity market where consumers can either buy power from a conventional retailer or through an aggregator who purchases power from the electricity market and then distributes it (for a subscription price) to his clients. In this article, we propose a coalition-based game-theoretic formulation of the problem, determine the game's core, and then provide a hierarchical learning algorithm for planning the aggregator's pricing strategy.

Extensions will be made by considering more general probabilistic models of error generation and characterizing the core and Shapley value under each model.

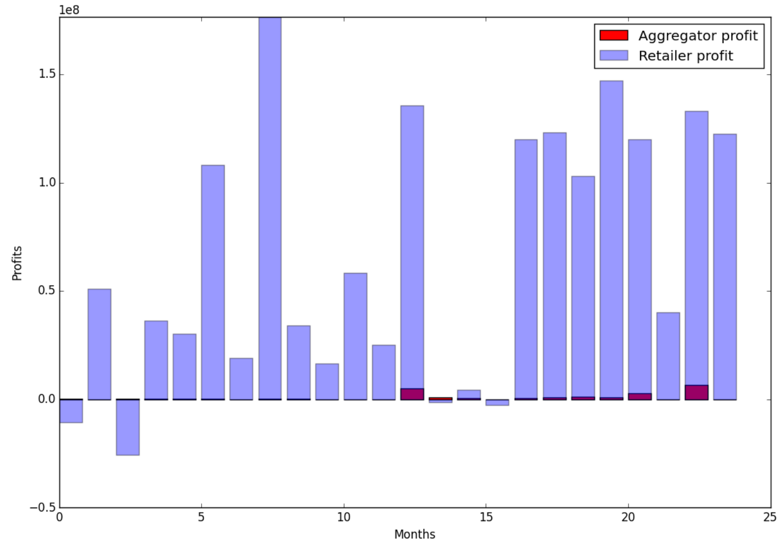
References

[1] Cesa-Bianchi N., Lugosi G., Prediction, Learning, And Games, Cambridge University Press, 2006

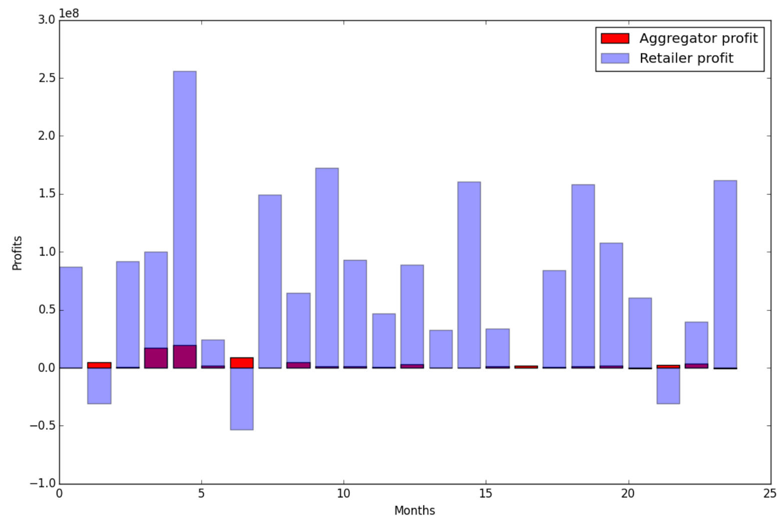
[2] Le Cadre H., Papavasiliou A., Smeers Y., Wind Farm Portfolio Optimization under Network Capacity Constraints, European Journal of Operational Research, volume 241, Issue 2, pages 560-574, 2015

[3] Meyn S., Barooah P., Busic A., Chen Y., Ehren J., Ancillary Services to the Grid using Intelligent Deferrable Loads, IEEE Transaction on Automatic Control, volume 60, Issue 11, pages 2847-2862, 2015

[4] Mochaourab R., Jorswieck E., Coalitional Games in MISO Interference Channels: Epsilon-Core and Coalition Structure Stable Set, IEEE Transactions on Signal Processing, volume 62, Issue 24, pages 6507-6520, 2014



(a)



(b)

Figure 1: We plot the dynamics of the providers' profit over 24 months for $\beta = 0.3$ in (a) and $\beta = 0.9$ in (b).

[5] Vasirani M., Ossowski S., Smart Consumer Load Balancing: State of the Art an an Empirical Evaluation in the Spanish Electricity Market, Artificial Intelligence Revue, volume 39, Issue 1, pages 81-95, 2013